PREDICTING THE UNSATURATED HYDRAULIC CONDUCTIVITY USING MULTI-POROSITY WATER RETENTION CURVES

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In order to better describe the water retention characteristic (WRC) of multi-modal soils, a multiporosity retention model is proposed based on a combination of several van Genuchten (VG) type subcurves. The multi-porosity WRC model keeps some of the desired properties of the VG-curve (continuously differentiable, strongly monotonous function with zero slopes at saturation and toward the dry end). Disadvantages of the multi-porosity curve are the increased number of model parameters (three for each additional subcurve), lack of a physical basis of the model parameters, and absence of a closed-form invertible function (forcing one to calculate the unsaturated hydraulic conductivity with numerical methods). The model accurately describes retention data of a variety of undisturbed soils with distinct bimodal or nonsymmetric pore-size distributions. Generally, there is a similarity between the shape of the multi-porosity retention and the predicted hydraulic conductivity function, \( K(h) \). \( K \)-predictions near saturation were found to be extremely sensitive to the shape of the fitted WRC curve in this region. Because the "natural" saturated water content is an ill-defined parameter, and because of hysteresis affecting the WRC in the wet range, the predicted \( K \) should not be matched with the measured saturated conductivity, but with a measured \( K \) value at some point less than saturation. The residual water content, \( \theta_r \), was found to have little or no effect on the predicted \( K \)-function in the wet region.

INTRODUCTION

An assessment of the behavior of pollutants and other dissolved chemicals in the vadose zone requires a quantitative evaluation of the governing water flow and solute transport processes. Improvements in the conceptual understanding of these processes in the unsaturated soil have lead to sophisticated numerical simulation models, which require accurate input and system parameters [Kool et al., 1987]. Of fundamental importance in describing the flow of water in the unsaturated zone is knowledge of two constitutive relationships: the water content versus pressure head relationship, \( \theta(h) \), and the dependence of the hydraulic conductivity, \( K \), on the pressure head, \( h \), or the water content, \( \theta \). In order to determine these relationships a large number of laboratory and field methods has been developed [for an inventory see Klute, 1986; Bruce and Luxmoore, 1986; Klute and Dirksen, 1986; Green et al., 1986]. Common to all measurement methods is a certain conflict between accuracy of the results and the necessary expense. The success of applying simulation models, particularly to heterogeneous fields, may well hinge on the ability to determine the necessary parameters in sufficient number and of acceptable accuracy.

The most difficult single parameter to obtain is the unsaturated conductivity. Direct measurement methods are costly, time consuming and, without exception, difficult to implement. Much effort has consequently been put into the development of indirect methods to estimate \( K \). Macroscopic prediction models estimate \( K \) from \( \theta \) based on an analogy between the micro- and the macroscale. They have a theoretical basis and generally contain only a small number of adjustable parameters. Important shortcomings of most macroscopic models is the neglect of the influence of a variable pore-
size geometry on the hydraulic conductivity [Childs and Collis-George, 1950], and the assumption of having a spherical pore geometry [Brooks and Corey, 1966]. Statistical prediction models have the advantage that they use information about the pore-size distribution of a soil. They predict \( K \) from the more easily measured water retention characteristic (WRC) and some factor allowing for a variable pore tortuosity [Childs and Collis George, 1950; Fatt and Dykstra, 1951; Burdine, 1953; Wylie and Gardner, 1958; Marshall, 1958; Millington and Quirk, 1961; Kunze et al., 1968; Mualem, 1976b; Mualem and Dagan, 1978; for a review see Mualem, 1986].

An essential requirement for any successful prediction of \( K \) by pore-size distribution models is an accurate description of the WRC over the whole range of pressure heads. The parameterized WRC function fitted to measured WRC data often represents the data fairly accurately. Widely used functions are given by Brooks and Corey [1964], Campbell [1974], and more recently by Vachaud et al. [1975] and van Genuchten [1980]. If the WRC is given by an analytic expression, the application of Burdine’s or Mualem’s model may lead to a closed-form equation for \( K \). A particularly attractive feature of the resulting equations is that they facilitate a simultaneous determination of all parameters by numerical solution of the inverse problem using suitable data from transient flow experiments [Zachmann et al., 1982; Hornung, 1983; Kool et al., 1985; Parker et al., 1985; Kool et al., 1987].

Predictions of \( K \) from WRC data are now commonly applied, the most popular method being the WRC model of van Genuchten (VG) combined with Mualem’s predictive model. The resulting set of closed-form equations for the constitutive relationships contains up to seven independent parameters. Experimental investigations have shown that this combination successfully describes the WRC and \( K \) characteristic, particularly for disturbed and sandy soils, provided the prediction is matched to at least one measured conductivity (often the saturated conductivity) [van Genuchten, 1980; van Genuchten and Nielsen, 1985; Schindler et al., 1985].

However, there are cases where the prediction does not appear to work well. Soils of this type include undisturbed loams and clays with bimodal or multimodal particle-size distributions (PSD’s) [Parker et al., 1985; van Genuchten, 1980; Othmer et al., 1991], morainic soils [Jacobsen, 1989], soils with well-developed secondary pore systems stemming from aggregation, and soils with a macropore system derived from roots and/or faunal action [Stephens and Rehfelt, 1985; Anderson and Cassel, 1986; Hantschel, 1987]. Schjønning [1985] investigated pore-size distributions of seven agricultural topsoils and found that morainic soils typically showed a "two-top" distribution curve. The PSD’s of these soils resulted in water retention characteristics which cannot be properly represented by the VG model. This limitation is also true for a series of similar S-shaped WRC models which essentially always represent "harmonic" unimodal PSD’s [van Genuchten and Nielsen, 1985].

Several studies [Stephens and Rehfelt, 1985; van Genuchten and Nielsen, 1985; Vogel and Cislerova, 1988; Sidironopulos and Yannopoulos, 1988] have reported that \( K \)-predictions by Mualem’s model are sensitive to the values of the residual water content, \( \theta_r \), and the saturated water content, \( \theta_s \). This raises the question whether or not there are systematical errors in the VG model for predicting \( K \) when the WRC is not adequately represented by the fitted retention curve.

The importance of an accurate description of the WRC may be evident also from its use in computational procedures, where \( K \) is calculated from the soil water diffusivity, \( D \), by
where $d\theta/dh$ is the slope of the WRC. Anderson and Cassel [1986] used this method for 450 undisturbed soil cores for which the diffusivity was determined by the hot air method [Arpa et al., 1975]. Also, the WRC is sometimes used to calculate water content changes of soil cores during outflow experiments. Hantschel et al. [1987] showed that application of the method of Becher [1971] resulted in $K$ values which were extremely sensitive to small changes in the slope of the WRC near saturation. The changes were even greater than those obtained with the predictions by Mualem's equation and, interestingly, went into two different directions: a higher slope $d\theta/dh$ resulted in higher calculated "Becher"-$K$ values but, because of a steeper $K(h)$-function, in lower predicted $K$ values [Durner, 1991]. It is therefore possible to adjust the value of the saturated water content in order to arrive at a good match between predicted and "measured" values.

The objective of this study was to develop a modified water retention model which describes the WRC of soils with multimodal pore systems, and to combine the retention model with Mualem's equation to predict $K$. Results for several soils will be compared with predictions obtained with the more commonly applied unimodal VG model.

**THEORY**

The WRC model used in this study is a weighted sum of van Genuchten's [1980] equation, further referred to as the MVG ("Multi-Van-Genuchten") model:

$$S_c = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \sum_{i=1}^{k} w_i \left[ \frac{1}{1 + (\alpha_i h)^n_i} \right]$$

(2)

where $S_c$ is the effective degree of saturation, $h$ is the pressure head (cm, for notational convenience assumed to be positive in unsaturated soil), $\theta$ is the volumetric water content (cm$^3$ cm$^{-3}$), $\theta_s$ and $\theta_r$ are the saturated and residual volumetric water contents (cm$^3$ cm$^{-3}$), respectively, $k$ is the model modality, $\alpha_i$ (cm$^{-1}$), $m_i$ and $n_i$ are VG parameters for each subcurve, and $w_i$ are weighing factors subject to the constraints $0 < w_i < 1$ and $\sum w_i = 1$. In this study, the model is always used with the constraints $n > 1$ and $m = 1 - 1/n$. The combination approach for the WRC has been independently proposed also by Reignier [1986] and Othmer et al. [1991].

Figure 1 depicts the construction of a dual MVG curve ($k=2$). The assumed pore-size distribution is shown in the Figure 1b. In this example, subcurve 1 is weighted with 0.4 and may reflect an interaggregate pore system, whereas subcurve 2 may represent the basic pore system of the soil matrix characterized by a more-or-less symmetric particle-size distribution.

The particular form of the MVG retention model, Eq. (2), was chosen because it is very flexible and keeps some of the desired properties of the original van Genuchten (VG) curve. The MVG model has a simple mathematical form, is smooth (continuous slope), has zero slopes at saturation and toward the dry end (which makes sense physically as pointed out by van Genuchten and Nielsen [1985]), and increases monotonously over the entire range of pressure heads. This last characteristic is also
Fig. 1. Schematic of (a) bimodal MVG soil water retention curve, and (b) associated pore-size distribution density, $d\theta/d(\log(h))$. Subcurve 1 has a weight of 0.4, and assumes $\alpha = 0.1$ and $n = 2.2$; subcurve 2 has a weight of 0.6, with $\alpha = 0.01, n = 1.4$.

a physical requirement. The maximum modality should in general be chosen ahead of the fitting procedure, thereby avoiding that the curve will follow the scatter of measured data in some uncontrollable manner. Although this is an obvious advantage as compared to simple spline interpolations, a similar behavior may be obtainable also by more sophisticated spline fittings subject to appropriate constraints. The MVG model can be easily modified by allowing $m$ to be an independent variable, or by keeping this parameter constant, in order to find more suitable shapes for a particular data set. A comparison of various interpolation schemes is discussed by Durner [1991].

Disadvantages of the curve are the increased number of parameters (three for each additional subcurve), and the lack of a closed-form inversion of the retention function. As for the unimodal case, the parameters are essentially fitting parameters with a limited physical meaning. Figure 1 indicates that the values $1/\alpha_i$ are related to the positions of local maxima of the pore-size distribution density, whereas the $n_i$ are related to their width. If the parameters $m_i$ are set to 1, $1/\alpha_i$ becomes the position of the inflection point of each subcurve.

The MVG curves were fitted to measured WRC data using a nonlinear least-squares fitting program based on the method of steepest descent. The data used in this study pertained to multiporous soils and were taken rather arbitrarily from the literature. When not available in tabular form, the data were retrieved by digitizing figures in the original publications.
The predictive equation for the hydraulic conductivity given by *Mualem* [1976b] is

\[
K_r = \frac{K}{K_s} = S_e \left[ \int_0^{s_e} \frac{dx}{h(x)} \right]^2 \int_0^1 \frac{dx}{h(x)}
\]

where \( t \) is an empirical constant allowing for a saturation-dependent pore tortuosity, \( K_s \) is the saturated hydraulic conductivity and \( K_r \) is the relative hydraulic conductivity. In this study \( t \) is set to 0.5 [Mualem, 1976b].

Since the MVG model is not invertible, \( K_s \) must be calculated numerically and, hence, the resulting \( K(\theta) \) and \( K(h) \) functions cannot not be given in a simple parameterized form. To facilitate their use in simulation models, it may be appropriate to fit the predicted curves again with suitable functions.

**RESULTS AND DISCUSSION**

*Fitted WRC's and Types of Modality*

The eight soils selected for this study cover a wide range in soil physical properties, while also showing a bimodal or multi-modal porosity. The basic physical properties of the soils are listed in Table 1. Figures 1a through 11a show for each soil the measured water retention data, the fitted VG and MVG curves \( \theta(h) \), and the associated pore-size distributions given by the derivative \( d\theta/d(\log(h)) \). The fitting parameters and the residual standard deviations (RSD) are given directly in the figures. The RSD was calculated as the root of the average square distance, disregarding reductions in the degrees of freedom.

The figures illustrate the multimodality of the different WRC's, the inability of the unimodal curve to describe most retention data sets, and the excellent fits obtained with the MVG-curves. The different modalities may be grouped into four different pore classes:

1. A primary (textural) pore system at intermediate pressure heads as determined by the particle-size distribution of the soil. The maximum for this pore class lies within a wide range, from pF = log(h) = 1 for coarse sand (Soil 1) up to more than 4 for clays (Soil 5). The width of this maximum is often very narrow for sands, and relatively wide for fine-textured soils. This pore system usually determines the general shape of the WRC.

2. A secondary (structural) pore system caused by aggregation or microchannels. The mode of this pore system may be located somewhere in the region 0.5<pF<2, depending upon the size and the genetic development of the aggregates (Soils 3, 4, 5, and 7). The water flux in this pore system still follows Darcy's law, while the representative elementary volume, REV [Bear, 1972], for this class of pores is small enough to still be included in the usual soil samples. The peak width is generally narrow to moderate.

3. Macropores, particularly those caused by faunal action and decaying plant root channels. For many soils, the existence of this pore class can be deduced from the large difference between the highest measured water content and the calculated or measured total porosity [e.g., Hantschel, 1987]. As will be shown later, macropores can cause an almost chaotic behavior of the \( K \)-predictions. This pore class is hardly
TABLE 1. Physical Properties of the Example Soils

<table>
<thead>
<tr>
<th>No.</th>
<th>Soil Name and Reference</th>
<th>$\rho_b$ (g cm$^{-3}$)</th>
<th>$\epsilon$ (%)</th>
<th>$\theta_s$ (%)</th>
<th>log($K_s$)</th>
<th>RSD from fitted curve</th>
<th>VG</th>
<th>2-MVG</th>
<th>3-MVG</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fine uniform Sand <em>Stephens and Rehfildt [1985]</em></td>
<td>1.58</td>
<td>40.2</td>
<td>36.2</td>
<td>-3.43</td>
<td>0.46</td>
<td>0.16</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Coarse Sandy Loam <em>Schjønning [1985]</em></td>
<td>1.55</td>
<td>41.3</td>
<td>40.3</td>
<td>-</td>
<td>1.08</td>
<td>0.13</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Portsmouth Sandy Loam <em>Anderson and Cassel [1986]</em></td>
<td>1.49</td>
<td>43.8†</td>
<td>39.1</td>
<td>-2.69‡</td>
<td>0.62</td>
<td>0.16</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Solar Village Clay <em>Hampton [1988]</em></td>
<td>1.30</td>
<td>52.7</td>
<td>55.6</td>
<td>-5.76</td>
<td>1.03</td>
<td>0.31</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Beit Netafa Clay <em>van Genuchten [1980]</em></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-8.01</td>
<td>0.27</td>
<td>0.06</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Roskilde Loam <em>Jacobsen [1989]</em></td>
<td>1.44</td>
<td>44.9</td>
<td>43.6</td>
<td>-4.43§</td>
<td>0.31</td>
<td>0.20</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Sandy Clay Loam <em>Parker et al. [1985]</em></td>
<td>1.53</td>
<td>42.3†</td>
<td>40.2</td>
<td>-5.95</td>
<td>0.39</td>
<td>0.14</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>G.E. No.2 Sand <em>van Genuchten and Nielsen [1985]</em></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.17</td>
<td>0.05</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

† Calculated assuming a particle density of 2.65 g cm$^{-3}$
‡ Geometric mean of 150 replicates (CV=3300)
§ Geometric mean of 18 replicates (SD=0.455)

accessible to experimental measurements since water retention is not accurately measurable at pressures between 0 and 1, while instrumental and sample-size dependent air entry problems may produce unrepeatable hysteresis phenomena. Any "peak" in this pore class, hence, cannot be defined accurately. Furthermore, the REV for this pore class is generally larger than the sample size. This makes its measurement in the laboratory for eventual use in field situations almost impossible. Finally, one may assume that flow in these pores is not, or at least not always, of the Darcian type [Beven and Germann, 1982].

4. A pore class in the very dry region above the highest pressure normally used in WRC measurements. The example of Hampton [personal communication, Soil 4] indicates that there may be no clear separation between textural pores (class 1) and this pore class. Few measured WRC data for this dry region have been published. If we assume that the water content of oven-dried soil is equivalent to a pressure head of 10$^7$ cm, it follows that each complete water retention curve must somehow interpolate between the lowest measured water content (as measured in pressure chambers, usually at pF = 4.2) and zero water content at pF = 7. This phenomenon is certainly not adequately represented by a curve which approaches a "residual water content," and is not able to predict any hydraulic conductivity below this value. van Genuchten [1980] addressed this problem and concluded that these changes "...would be inconsistent with the general shape of the $\theta(h)$ curve defined by the VG equation, and probably invalidate the concept of a residual soil water itself." One may be further question if water loss in this region can be interpreted as being caused by pores which empty at the equivalent capillary pressure. Instead, the water
Fig. 2. Measured and predicted VG (dotted lines) and MVG (solid lines) hydraulic properties for Fine Uniform Sand (Soil 1): (a) measured (data points) and fitted water retention, $\theta(h)$, and associated pore-size distribution, $d\theta/d(\log(h))$, curves, (b) predicted hydraulic conductivity functions, $K_c(h)$, and (c) predicted conductivity functions $K_c(\theta)$. The saturated water content was equated to the highest measured water content.

Fig. 3. Same as Figure 2, for Coarse Sandy Loam (Soil 2) from Ronhaeve, Denmark (morainic soil). The saturated water content equated to the highest measured water content.
SANDY LOAM, Btg

[Anderson and Cassel, 1986]

\[ \begin{align*}
\theta_s &= 0.391 \\
\theta_r &= 0.0 \\
\alpha &= 0.0444 \\
\text{RSD} &= 0.62\% \\
\end{align*} \]

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3-modal MVG

\[ \begin{align*}
\theta_s &= 0.391 \\
w &= 0.50 \\
\theta_r &= 0.0 \\
\alpha &= 0.827 \\
\sigma &= 0.0177 \\
\text{RSD} &= 0.02\% \\
\eta &= 1.102 \\
\end{align*} \]

Fig. 4. Same as Figure 2, for Portsmouth Sandy Loam (Soil 3). The saturated water content was equated to the highest measured water content. All measured values are averages of 150 soil cores.

CLAY AGGREGATES

[Hampton, 1988]

\[ \begin{align*}
\theta_s &= 0.556 \\
\theta_r &= 0.036 \\
\alpha &= 0.0208 \\
\text{RSD} &= 1.03\% \\
\eta &= 1.206 \\
\end{align*} \]

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2-modal MVG

\[ \begin{align*}
\theta_s &= 0.556 \\
w &= 0.50 \\
\theta_r &= 0.0 \\
\alpha &= 0.0208 \\
\sigma &= 0.00002 \\
\text{RSD} &= 0.31\% \\
\eta &= 1.626 \\
\end{align*} \]

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3-modal MVG

\[ \begin{align*}
\theta_s &= 0.556 \\
w &= 0.20 \\
\theta_r &= 0.0 \\
\alpha &= 0.0138 \\
\sigma &= 0.0244 \\
\text{RSD} &= 0.08\% \\
\eta &= 5.251 \\
\end{align*} \]

Fig. 5. Same as Figure 2, for packed Solar Village Clay (Soil 4). Data from Hampton [personal communication].
loss may be due to some thinning of water films on soil matrix surfaces and at particle contact points [Scheffer and Schachtschabel, 1988].

Predictions of the hydraulic conductivity in the extreme dry region may be suspect for three reasons: (i) water flow in this "pore class" may no longer follow Darcy's law [Verruijt, 1981; Harder and Blümel, 1987]; (ii) when a soil is so dry, overall water flow may become dominated by flow in the vapor phase, particularly if there are temperature gradients [Rose, 1968]; and (iii) the capillary bundle concept used in the statistical prediction models may be inappropriate for this region. This pore class can probably be neglected for most field problems.

There are certainly no clear borders between the above theoretically defined soil pore classes. It is often impossible to define an exact pressure head where the macropore system begins (Soils 2, 3). The prediction of \( K \) is further complicated by entrapped air and hysteresis, both of which lead to a difference between the total pore volume and "natural" water saturation, as well as by swelling of nonrigid soils. Apart from trying to attribute each of the observed PSD maxima to one of the basic pore classes, one may find even in disturbed soils that mixing of different particle sizes causes pore-size distributions which are not symmetric and, hence, cannot be adequately represented by an unimodal WRC (Soil 2).

\( K(h) \) Predictions

Figures 2b through 11b show the predicted relative hydraulic conductivities \( K \), for the uni- and multi-modal cases. A common observation, confirmed by other examples not further shown here, is the similarity of the wet parts of the WRC and predicted \( K(h) \) functions. This similarity is physically reasonable: if a soil looses much water in a certain pF-interval (i.e., because of a steep local slope \( d\theta/dh \)), the large number of draining pores in that interval should cause a corresponding steep decrease in the hydraulic conductivity function. The predominant factor causing differences between the unimodal and multimodal predictions is the slope of the water retention curve near saturation. For clays, but also for certain coarse soils (not further documented here), the MVG curve in this region lies between the Brooks and Corey curve and the VG model, thus representing a "sharper" air entry region. The VG predicted conductivity equation in this region decreases too abruptly to lower values, whereas the MVG predictions for \( K \), keep values which are much closer to unity (Soils 4 and 5). The more the measured water content at the air entry value is underestimated (smoothed out) by the VG model, the more will the resulting \( K \)-prediction be underestimated. The maximum differences seem to occur in the ecologically important interval \( 1<pF<2 \), and may easily reach several orders of magnitude, even for relatively small water content differences over this pF interval. Sometimes the predicted \( K(h) \) curves were found to approach each other again in the dryer region.

For loamy soils (Soils 3, 7) and morainic soils (Soils 2, 6), as well as for most soils with macropores in the wet region (Soils 1, 3), the situation is different. The PSD of these types of soil are often broad, multimodal or not symmetric. The fitted VG curve for these soils cannot represent the water loss close to saturation. Consequently, the predicted VG conductivity remains longer near saturation relative to the MVG prediction, as illustrated by Figures 2, 3 and 4. Again, the \( K(h) \) function predicted with the VG and MVG curves may approach each other in the dryer region.
Fig. 6. Same as Figure 2, for Beit Netofa Clay (Soil 5). The saturated water content was fitted in order to get a better fit with the measured conductivity data.

Fig. 7. Sensitivity of $K$-predictions to the residual water content for a morainic soil from Roskilde, Denmark (Soil 6). Residual water contents were set to 0 and 0.15 for the VG model (top plots), and 0 and 0.12 for the MVG model (bottom plots).
The only possible way to assess the practical relevance of these different results is to compare the predicted curves with measured conductivities. Figure 6 shows this comparison for Beit Netofa Clay for which K-data are available. The MVG predicted conductivities in this example were much closer to the measured values. However, it has to be mentioned that the saturated water content in this example was not taken as the highest measured water content (at pF = 2), but was set about 2% higher in order to achieve a better prediction of the observed conductivities. Whereas the assumption of having a higher water content appears reasonable, it still remains to be tested if the MVG model generally leads to better K-predictions. A series of comparisons of multimodal WRC predictions with measured values may provide better answers. It may also be worthwhile to test if the MVG model would lead to better predictions in cases where the Mualem prediction has been shown to give poor results.

*K*(θ) Predictions

Because of the strong hysteretical nature of the WRC, it is often recognized that conductivities in simulation models should be taken from the *K*(θ) function [Mualem, 1986]. Figure 2c to 11c show these functions for the example soils. As expected, the most serious differences in the predictions occur in the extremely dry region when the fitted residual water contents become different. The differences approach infinity as soon as θ, is reached (*K*(θ) is no longer defined below this value). Hence, trying to use K-predictions in very dry soils is only reasonable when the residual water content equals zero.

The observed differences in the *K*(h) curves are reflected to some extent also in the *K*(θ) functions, although the width of the affected interval depends on the slope of the WRC over that region. When the *K*(h) predictions based on the MVG and VG models differ greatly over an interval where the slope of the WRC is steep, even if that interval is relatively narrow, then the difference will appear over a relatively broad region in the *K*(θ) plot (e.g., Soil 1).

The Role of θ

There is no a priori reason that θ, should influence K-predictions near saturation in a noticeable manner. van Genuchten [1980] concluded from his studies that the influence of θ, on K-predictions is small as long as the general fit of the WRC data is good. He recommended to take θ, equal to a measured water content at large values of h. Other studies [Stephens and Rehfeldt, 1985] showed an effect of θ, on the K-predictions which may be explained indirectly: assuming a higher θ, value always leads to a higher n value in the VG curve. This, in turn, leads to higher conductivities in the wet region, but a steeper curve at intermediate pressure heads. Because of the weighted combination of two subcurves, the MVG model shows no direct effect of θ, on the slope near saturation. It is therefore expected that θ, plays no role in predicting K in the wet region. This is illustrated in Figure 7 (top plots) which shows two VG curves with different residual water contents fitted to WRC data of a morainic soil [data by Jacobsen, 1989]. The same data were also fitted with the MVG model (lower plots in Figure 7b). The MVG predictions in the wet region were found to be insensitive to a change in θ, from 0% to 12%.

Consequently, when the predicted conductivity values are of importance only within the wet region, one may delete measured data from the WRC data set in order to improve the VG fit. Figure 8 shows the result of fitting the VG and MVG curves to the
COARSE SANDY LOAM

[Schjonning, 1985]

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1-modal VG

\[ \theta_s = 0.403 \]

\[ \theta_r = 0.0 \]

\[ \sigma = 0.444 \]

\[ \text{RSD} = 0.14\% \; n = 1.062 \]

2-modal MVG

\[ \theta_s = 0.403 \quad w = 0.30 \quad 0.70 \]

\[ \theta_r = 0.0 \quad \sigma = 0.686 \quad 0.0059 \]

\[ \text{RSD} = 0.04\% \; n = 1.213 \; 1.077 \]

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Fig. 8. Same as Figure 3, but ignoring the lowest measured water content data point (shown between brackets). The \( K_r(h) \) plot also shows the three-modal prediction.

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FINE SAND

[Stephens, 1986]

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1-modal VG

\[ \theta_s = 0.400 \]

\[ \theta_r = 0.054 \]

\[ \sigma = 0.0306 \]

\[ \text{RSD} = 0.94\% \; n = 2.809 \]

2-modal MVG

\[ \theta_s = 0.400 \quad w = 0.72 \quad 0.28 \]

\[ \theta_r = 0.070 \quad \sigma = 0.0233 \quad 8.846 \]

\[ \text{RSD} = 0.15\% \; n = 6.224 \; 1.243 \]

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Fig. 9. Same as Figure 2, but with the saturated water content set equal to the porosity.
data of Soil 2 while ignoring the lowest measured water content data point. The VG fit dramatically improved the predicted K-curves, which are now much steeper than in Figure 3, and much closer to the MVG curves. Ignoring data points in the dry region also improved the VG prediction for Beit Netofa Clay (results not shown here).

**Macropores, Air Entrapment, and the Definition of "Saturation"**

Considering the important effect of the slope of the WRC near saturation, the question arises as to the exact role of the saturated water content, \( \theta_s \), in the K-predictions. Some studies addressing this problem have recommended to use the measured water content closest to saturation ("natural" saturation), rather than the total porosity, as a more appropriate point for matching the fitted WRC [Stephens and Rehfeldt, 1985; Hantschel et al., 1987; Vogel and Cistierova, 1988]. The study of Hantschel et al. [1987] showed an improved agreement of predicted and "Becher"-calculated conductivities when \( \theta_s \) was set equal to the water content at 2 cm pressure head, rather than to the total pore volume. The problems seem to arise partially from the inability of the VG curve to fit data accurately when the WRC bends downward close to the saturation point (Figure 2). Hence, forcing the VG to the total porosity value gives a worse overall fit.

Theoretical considerations suggest that it is important to account for the existence of macropores, which lead to a bend of the WRC curve close to saturation, when applying a statistical-predictive model for K. This is, at least in principle, possible with the MVG model. Still, a number of problems remain. For example, as addressed above, there are no precise measurements possible in the wet region. From a mathematical point of view, the parameters of the subcurve representing this part of the MVG curve may be difficult to determine. This means that the exact position of the MVG equation near saturation may be somehow arbitrary, even though its influence on the predicted conductivity is enormous. This is illustrated in Figure 9 which, in contrast to Figure 2, has a \( \theta_s \) set equal to the value of the total porosity (0.4 instead of 0.36). The MVG equation fits the data again quite adequately, but now the predicted hydraulic conductivity drops by four orders of magnitude within the first 10 cm of pressure head. Arbitrary manipulations of \( 1/\alpha \) to lower pressures (which do not alter the fitting result, since there are no data in this region) may increase this difference to any value.

The ill-defined nature of the WRC in the macropore region is stressed by another example, using data from King [1965], taken from van Genuchten and Nielsen [1985]. The VG predicted curve in this case overestimated the measured K-values (Figure 10), while the MVG curve resulted in close agreement with the measured values. The only difference between the two curves is due to adding a subcurve with a weighing factor of 0.01, and altering the saturated water content by 0.002! This modification, which is far below any measurement precision, was obtained by adding a few artificial retention data close to saturation.

**The Dominance of Initial Slope of the WRC**

Figure 11 suggests that the MVG should not be uncritically adopted for predictive purposes. Water retention data of Parker et al. [1985] were fitted with the 1-, 2- and 3-modal MVG. Figure 11a shows some improvement in the retention fit using the 2-modal curve as compared to the VG equation, leading to a much higher K prediction around pF = 1.5. However, a close look at the retention data in the sensitive wet region shows that water loss is almost linear up to a pF of 1.8, and less well described by the 2-modal model. Application of the 3-modal MVG equation reflects a very similar PSD,
Fig. 10. Sensitivity of $K_r$ to the saturated water content (G.E. No. 2 Sand, Soil 8). A small change in the saturated water content leads to a sudden drop in the hydraulic conductivity near saturation.

Fig. 11. Sensitivity of $K_r$, predictions to the slope of the retention curve near saturation for a Sandy Clay Loam (Soil 7).
Multi-porosity Water Retention Curves

but with subtle yet important differences in the wet region. The resulting prediction is in extreme contrast to the 2-modal curve, differing by up to three orders of magnitude in the wet region. The VG prediction was located between the other curves. It appears impossible to judge the reliability of the predictions in this case since the small retention differences causing the observed shifts may well lay within the measurement error. Furthermore, averaging over single measurements may lead to a misleading shape of the curve.

Considering that the saturated conductivity is normally measured at the "natural" water content rather than at the absolute saturated water content, that air entrapment in the soil often impedes water flow, that flow in macropores may not follow Darcy’s law, and that the macropore system can be extremely heterogeneous spatially, it should be evident that it is even less appropriate to match the MVG predicted $K$-functions to a saturated conductivity value, as is the case for the VG equation [Green and Corey, 1971; Jacobsen, 1989]. Instead, in accordance with van Genuchten and Nielsen [1985] and Vogel and Cistlerova [1988], it is recommended to match the theoretical curves with measured data at some unsaturated value. A future study of considerable interest would be to match a series of predicted MVG $K$-curves from a large number of "replicate" soil cores at unsaturated conditions, and subsequently predict the variability of the saturated conductivity of a field soil.

Parameter Estimation Using Inverse Methods

One potentially powerful method for determining all hydraulic parameters simultaneously from one soil sample is to apply a numerical parameter inversion method to some type of transient flow experiment. Important to acquiring valid results is the correct parameterization of the constitutive relationships. Apart from cases where there is an obvious inconsistency in the observed results, there is no a priori way of deciding from, for example, a one-step outflow experiment if the soil has a multi-modal pore system. It is important to consider that, even when an incorrect model for the soil hydraulic properties is used, it may still be possible to obtain an apparently acceptable solution of the inverse problem [Zachman et al., 1982]. However, the hydraulic functions corresponding to the solution may be erroneous. Future research should therefore investigate if a multistep procedure (with steps not necessarily going to equilibrium) may lead to better results with less expense in time and costs, while avoiding difficulties in the experimental implementation.

CONCLUSIONS

The MVG curve successfully described the water retention characteristics of several soils having multimodal pore-size distributions. Coupling the model with the predictive hydraulic conductivity model of Mualem can, for some soils, lead to radically different $K(h)$ and $K(\theta)$ curves as compared to the unimodal van Genuchten model, even for visibly very small differences in the fitted retention curves. A general similarity was found between the shape of the wet part of the WRC and the predicted $K(h)$ function. In loamy and morainic soils, the slope $d\theta/dh$ of the unimodal VG curve near saturation is generally steeper than the slope of the MVG curve, thus leading to an overprediction $K$ in the wet range. In soils with a distinct air entry value, the unimodal VG curve has a tendency to underestimate the hydraulic conductivity, the maximum difference being in the ecological important pF range between 1 to 2. The residual water content in the
MVG function was found to have no direct influence on the K-predictions in the wet region. The sensitivity of predicted K's to changes in the shape of the WRC close to saturation was found to be so extreme that predictions in this region should be interpreted with care. The predicted K(h)-function consequently should not be matched with the measured saturated conductivity, but with some unsaturated value. Further tests are needed to discern if K-predictions with the MVG curves are more accurate, and if they lead to significant improvement for field situations. The hysteretic WRC and the saturated and unsaturated hydraulic conductivities should always be determined on the same sample. Parameter estimation by numerically solving the inverse problem using multistep outflow experiments may be the optimal approach, and should be further developed.

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