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Estimation of van Genuchten-Mualem Parameters from Spectral Induced Polarization Measurements

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SUMMARY

The application of spectral induced polarization (SIP) surveys to estimate hydrological properties of the subsurface is promising. For the interpretation of the SIP data, the knowledge of the relationships between soil hydraulic properties and parameters obtained from SIP measurements is essential. Therefore, we performed hydrological and SIP measurements in the laboratory on seven natural soils and a medium sand sample with a narrow grain size distribution. The SIP spectra were interpreted with the Debye decomposition, which yields integrating parameters for the resulting relaxation time distribution. To derive soil hydraulic parameters from the hydrological experiments, we fitted the van Genuchten-Mualem model, which is widespread in soil physics, to the measured hydrological data. In this study, we concentrate on the relationship between two parameters: the normalized chargeability from the Debye decomposition of SIP spectra and the parameter alpha from the van Genuchten-Mualem model. We present an approach to estimate the parameter alpha from the normalized chargeability.
Introduction

Spectral induced polarization (SIP) measurements can be a useful tool to estimate the hydraulic conductivity of the subsurface (e. g., Hördt et al., 2009; Breede et al., 2011). To identify relationships between complex geoelectrical parameters and soil hydraulic properties, studies often focus on sand or sand-clay mixtures (e. g., Breede et al., 2012). We performed SIP and hydrological laboratory measurements on soil samples from different locations in the vicinity of Braunschweig, Germany. The SIP spectra are interpreted with the Debye decomposition, which yields integrating parameters to summarize the resulting relaxation time distribution. From the hydrological experiments characteristic parameters are derived by fitting the van Genuchten-Mualem model, which is widespread in soil physics. For this study we limit to the correlation between two parameters, the normalized chargeability resulting from the Debye decomposition of SIP spectra and the parameter $\alpha$ from the van Genuchten-Mualem model.

Methods

SIP is a geoelectrical method, where the complex resistivity is measured at several frequencies, usually between 1 mHz and 1 kHz. For our study, we confine the frequency range between 10 mHz and 100 Hz, because measuring at frequencies lower than 10 mHz is very time consuming and the investigation of frequencies higher than 100 Hz is interfered by electromagnetic coupling effects. The resulting amplitude and phase angle spectra of the complex resistivity are interpreted with the Debye decomposition presented by Nordsiek and Weller (2008), where the measured complex resistivity $\rho(\omega)$ is interpreted as the result of a superposition of $n$ Debye relaxation processes with relaxation times $\tau_k$ and chargeability values $m_k$:

$$\rho(\omega) = \rho_0 \cdot \left(1 - \sum_{k=1}^{n} m_k \cdot \left(1 - \frac{1}{1+i\omega\tau_k}\right)\right),$$  

with the DC resistivity $\rho_0$, the angular frequency $\omega$ and the imaginary unit $i = \sqrt{-1}$. The resulting relaxation time distribution $(\tau_k, m_k)$ is summarized by integrating parameters. One of these parameters is the total chargeability

$$m_{\text{total}} = \sum_{k=1}^{n} m_k.$$  

Weller et al. (2010) suggested to use the normalized chargeability $m_n$, which is the ratio of the total chargeability $m_{\text{total}}$ to the DC resistivity $\rho_0$.

We conducted the SIP measurements on samples saturated with calcium chloride solutions of different ionic strengths, varying from 0.001 to 0.02 mol/l. After finishing all SIP measurements, the Multi-Step Outflow (MSO) experiments were performed to determine the soil hydraulic properties. For the MSO experiments, a negative pressure head was applied to the bottom of the initially saturated soil column and was decreased step-wise. The volume of water flowing out of the sample was measured. The water retention curve and the hydraulic conductivity curve, were obtained as function of the pressure head. As an alternative to the MSO experiments, in some cases an evaporation experiment with the commercial HYPROP device by UMS GmbH (Munich) was used to determine the soil hydraulic properties. Instead of applying a decreasing pressure head to the samples, this method utilizes the evaporation of the water from the pore space of an initially saturated sample. A detailed description can be found in Peters and Durner (2008). The van Genuchten-Mualem relation (Mualem, 1976; van Genuchten, 1980) describes the hydraulic conductivity

$$K(S_e) = K_s \cdot S_e^l \cdot \left(1 - \left(1 - S_e^{1/m}\right)^m\right)^2$$  

with the effective saturation $S_e(h) = (1 + \alpha|h|^{n})^{-m}$, which depends on the pressure head $h$. $K_s$ is the saturated hydraulic conductivity and $\alpha$, $l$, $m$, and $n$ are empirical parameters. Fitting this relation to the results of the MSO and HYPROP measurements,
respectively, yields characteristic soil hydraulic parameters. One of these parameters, $\alpha$, is related to the inverse of the air entry pressure of the soil (e.g., van Genuchten, 1980) and therefore to the characteristic radius of the largest pores.

We built a sample holder that can be used for both, SIP and MSO measurements. Thus, it was not necessary to repack the samples between the electrical and hydrological experiments, which is critical for unconsolidated sediments as demonstrated by Bairlein et al. (2013).

**Material**

To study the relationship between the normalized chargeability $m_n$ and the van Genuchten-Mualem parameter $\alpha$, we investigated the materials listed in Figure 1 with SIP measurements and MSO and HYPROP experiments, respectively.

![Figure 1: The grain size distribution of seven soil samples. Three loam samples from Groß Gleidingen, Wolfenbüttel, and Völkenrode and four sand samples from Vollhüttel and Schunteraue. Additionally, we investigated a laboratory sand (not shown in this figure) with a narrow grain size range from 0.4 to 0.8 mm.](image)

**Results**

In Figure 2, we merged the results of SIP measurements at different ionic strengths of the saturating fluid and found that the normalized chargeability $m_n$ decreases with increasing $\alpha$. Assuming a linear relationship between $\alpha$ and the inverse of $m_n$ yields $\alpha_{SIP} = b \cdot (m_n)^{-1}$ with $b = 1.12 \cdot 10^{-4}$ S/m$^2$ and a correlation coefficient of $R = 0.48$ for the estimation of the parameter $\alpha$.

Weller et al. (2011) found a correlation between the normalized chargeability $m_n$ and the pore surface to volume ratio $S_{por}$:

$$S_{por} = C_F \cdot \frac{\sigma_f}{\sigma_w} \frac{m_n(\sigma_w)}{c_s(\sigma_f)},$$

with $\sigma_f$ as the fluid conductivity at which the empirical factor $c_s$ is determined and $\sigma_w$ as the fluid conductivity at which $m_n$ is determined. $C_F$ is a correction factor which is set to $C_F \approx 2$ for calcium chloride solution. $S_{por}$ is linearly related to the inverse of the pore radius (e.g., Zisser et al., 2010).
Figure 2: The normalized chargeability $m_\text{nl}$ compared to the parameter $\alpha_{\text{MSO}}$ resulting from MSO and HYPROP experiments, respectively. The solid line indicates $m_\text{nl} = b \cdot (\alpha_{\text{MSO}})^{-1}$ with the empirical factor $b = 1.12 \cdot 10^{-4} \text{ S/m}^2$. Open symbols mark samples where only SIP measurements were performed and $\alpha_{\text{MSO}}$ was adopted from another sample of the same material. These samples were not considered for the correlation.

Considering a linear relationship between $\alpha$ and the pore radius, eq. 4 can be transformed to:

$$\alpha_{\text{SIP}} = \frac{c}{2} \cdot \frac{\sigma_w}{\sigma_f} \cdot \frac{1}{m_\text{nl}(\sigma_w)}$$

(5)

where $c$ is an empirical factor. From our dataset, we choose 20 SIP measurements of five different samples saturated with calcium chloride solution with a conductivity of $\sigma_f = 110 \text{ mS/m (±10 mS/m)}$ to determine the factor $c = 2.4 \cdot 10^{-4} \text{ S/m}^2$. In Figure 3, the parameter $\alpha_{\text{SIP}}$ calculated with equation 5 is shown versus the parameter $\alpha_{\text{MSO}}$ determined from the hydrological experiments.

Figure 3: Comparison of $\alpha_{\text{SIP}}$ derived from SIP measurements and $\alpha_{\text{MSO}}$ determined with MSO and HYPROP experiments, respectively. The solid line indicates the value of $\alpha_{\text{MSO}}$. Considering only the samples where both SIP and hydrological results are available (filled symbols) yields a correlation coefficient of $R = 0.73$. 

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Conclusions

We investigated eight samples of unconsolidated sediments with SIP and the MSO/HYPROP method. The resulting complex resistivity spectra and the hydraulic conductivity curves were parameterized by the Debye decomposition approach and the van Genuchten-Mualem model, respectively. The parameter $\alpha$ was estimated with the normalized chargeability $m_n$ resulting from the Debye decomposition of the SIP spectra. The estimation of $\alpha$ can be improved when the electrical conductivity of the pore fluid is considered.

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References


