The use of flux-density measurements to improve the inverse determination of saturated hydraulic conductivity from multistep outflow experiments

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Problem
Multi-step outflow experiments in combination with inverse methods are a flexible tool for the identification of the soil hydraulic parameters (Hopmans et al. 2002). In this approach, usually time series of observations of cumulative outflow and tensiometer readings from a soil core are matched by numerical simulation by minimizing a suitable objective function. However, identifiability problems are reported which lead to considerable uncertainty in the estimates, in particular for the saturated hydraulic conductivity ($K_s$).

The objective of this study was to evaluate whether the inverse estimation of $K_s$ from multi-step outflow experiments can be improved by including flux density data, which can easily be inferred from measured outflow data, in the objective function.

Approach
The work was carried out with synthetically generated data, corrupted with noise. Based on these data, the response surface of the objective function was examined in the multidimensional parameter space. This was achieved by calculating the mean square error between model simulations and observations at a multiple of points from the parameter space for the three data types separately.

Materials and Methods

Governing Equations
One dimensional single-phase fluid flow in variably saturated porous media is described by the Richards equation:

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial z} \left[ K(h) \left( \frac{\partial h}{\partial z} \right) \right] = 0 \quad [1]$$

with:
- $h$: matric head
- $K(h)$: hydraulic conductivity
- $C(h)$: soil water capacity

The Mualem-van Genuchten model of the soil water retention curve and the unsaturated hydraulic conductivity function are given by (van Genuchten, 1980):

$$m(\theta) = \alpha \left[ \theta^\beta - 1 \right]^\frac{1}{\beta}$$  
$$K'(\theta) = K_s \theta^{n-1} \left[ 1 - (1-\theta)\theta^n \right]^\frac{1}{n}$$  

where $\theta$ is the normalized water content, $K_s$ is the saturated hydraulic conductivity, and $\alpha$, $n$ and $\tau$ are shaping parameters.

Initial and boundary conditions
A soil column of 10 cm length was assumed to be initially saturated with a hydrostatic pressure distribution. The upper boundary condition was zero flux, the lower boundary condition was given as a time-dependent Dirichlet condition specified in the matrix head described by a step function (Fig. 1).

Generation of synthetic data sets
Synthetic outflow and tensiometric data were generated by numerically solving equation [1] by use of the ESHPM model developed by Zurmühl (1994), for the following parameter values of the hydraulic functions:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>0.04 cm$^{-1}$</td>
</tr>
<tr>
<td>$n$</td>
<td>2</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.5</td>
</tr>
<tr>
<td>$K_s$</td>
<td>10 cm hr$^{-1}$</td>
</tr>
</tbody>
</table>

The synthetic data were corrupted with normally distributed noise ($\sigma_\theta=2.5 \times 10^{-3} \text{cm}$, $\sigma_h=0.5 \text{cm}$), in order to reflect random measurement errors.

Generation of response surfaces
The parameter space was confined to four dimensions by only examining the effect of the parameters $\alpha$, $n$, $\theta_s$, $K_s$. Two-dimensional response surfaces were calculated by keeping two parameters at their known true values and systematically varying the remaining two at a rectangular grid. Analysis was carried out separately for all three data types. Results are shown in Fig. 2.

Results

Conclusions
- The parameter response surfaces obtained from using the flux density data show greatly enhanced sensitivity to $K_s$, as compared to those obtained from the cumulative outflow and the matric head data.
- As flux density data can easily be computed from measured cumulative outflow data, their consideration seems to be a cost-effective way to improve the estimate of the conductivity function.
- Aggregating the three data types bears the potential to improve the poseability of the inverse parameter estimation problem. However, making this potential accessible relies on an appropriate weighting scheme when the data types are used within a multi-objective optimisation framework.

References

